

Unloaded Q -Factor of Stepped-Impedance Resonators

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Abstract—The paper presents general expressions for the unloaded Q -factor of stepped-impedance resonators partially loaded with high-dielectric-constant ceramics to realize miniaturized microwave bandpass filters. Some theoretical calculations for coaxial resonators are also presented in diagrams to show a correct design optimization of coaxial miniaturized filters. The formulas presented in this paper take into account the imaginary component of the characteristic impedance Z of the lines constituting the resonator; the results previously presented by other authors neglected such an imaginary component of Z .

The relevant influence that this imaginary component has for the calculation of the correct unloaded Q of the resonator is pointed out by this paper through a comparison between correct calculations and calculations performed by neglecting the imaginary component of Z . Some experimental results are compared with theoretical calculations.

I. INTRODUCTION

COMPACT BANDPASS microwave filters using coupled coaxial resonators in stepped-impedance structures partially loaded with high-dielectric-constant ceramics have been proposed in [1]–[4].

The price paid for miniaturization is a deterioration of the resonator quality (e.g., of the unloaded Q -factor, Q_0 , and of the frequency stability). Typically, the resonator length may be reduced 20–30 percent of a standard quarter-wavelength uniform impedance air-filled resonator with only a small increase of filter losses.

A discussion on the optimization of resonator Q_0 and therefore of filter losses is found in [2]. However, as this paper shows, the results of calculations published in [2] are not correct. The reason is that for a correct evaluation of Q_0 the small imaginary component of the characteristic impedance of the lossy lines cannot be neglected. As a consequence, the design optimization presented here yields very different results from the curves published in [2]. A general first-order analysis of the Q_0 evaluation is supplied in Section II by giving closed formulas for the various loss contributions to Q_0 ; a comparison with the results published in [2] is presented in Section III. Some experimental results are also presented and compared with the theoretical calculations in Section IV.

Manuscript received January 3, 1986; revised April 18, 1986.

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IEEE Log Number 8610506.

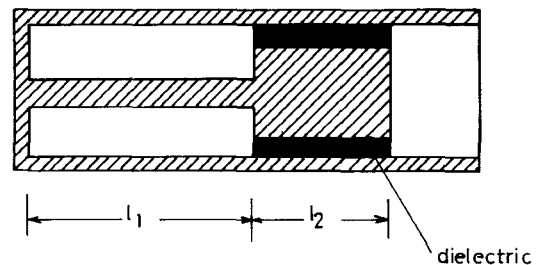


Fig. 1. Structure of a stepped-impedance resonator having a coaxial structure with uniform outer diameter.

II. ANALYSIS OF THE RESONATOR UNLOADED Q -FACTOR

The stepped-impedance resonator is shown in Fig. 1. It consists of two sections of length l_1 and l_2 , respectively, having characteristic impedance Z_1 and Z_2 with $K = Z_2/Z_1 < 1$. The section of length l_2 may be advantageously dielectric-loaded in order to obtain a stronger reduction both of l_2 and Z_2 and therefore of the overall length $l = l_1 + l_2$. The length reduction of the resonator with respect to the air-filled conventional uniform-impedance $\lambda/4$ resonator is discussed in [1]–[4]. A reduction factor can be defined as the following normalized length:

$$x = 4l/\lambda \quad (1)$$

which evidences the amount of the resonator length reduction with respect to the $\lambda/4$ air-filled resonator. If we neglect, in a first-order approximation (as in [2]), the effect of higher order modes in the vicinity of the impedance jump and at the open end and the losses in the radial surface of the jump, the unloaded Q -factor Q_0 depends on three main contributions which, for the sake of simplicity, may be evaluated separately

$$Q_0 = \frac{1}{1/Q_{0o} + 1/Q_{0d} + 1/Q_{0s}} \quad (2)$$

where Q_{0o} is the contribution due to the losses in the conductors of the transmission lines constituting the resonator, Q_{0d} resulting from the dielectric losses of section l_2 if it is dielectric-loaded, and Q_{0s} that comes from the losses in the terminal short circuit.

In the evaluation of the partial Q -factors (Q_{0o} , Q_{0d} , Q_{0s}) of (2), only the various lost powers have been separated, while the total magnetic and electric energies, stored in the

whole resonator, have been taken into account. In the first-order approximation calculations of this paper, these stored energies have been evaluated as in the lossless case.

A separate evaluation of the various loss contributions leads to a closed-form expression for Q_0 which is justified when the losses are small. Q_{0o} depends on the line attenuation constants, α_1 and α_2 of l_1 and l_2 , respectively, due to the ohmic losses in the line conductors, Q_{0d} on the dielectric loss factor $\tan \delta$, and Q_{0s} on the value of resistor R , which represents the lossy short circuit. Resistor R is given by the general expression which does not depend on the conductors configuration [5]

$$R = \frac{R_s}{Z_{oo}} Z_1 \quad (3)$$

where Z_{oo} is the wave impedance of the medium

$$Z_{oo} = \sqrt{\frac{\mu_o}{\epsilon_o}} \quad (4)$$

and R_s is the surface resistance of a conductor of conductivity σ_c

$$R_s = \sqrt{\frac{\omega \mu_o}{2 \sigma_c}} \quad (5)$$

The value of Q_{0o} can be evaluated by considering contributions Q_{01} and Q_{02} separately to Q_{0o} owing to the losses of the resonator sections l_1 and l_2 , respectively (by supposing that the dielectric, loading l_2 , is lossless)

$$Q_{0o} = \frac{1}{\frac{1}{Q_{01}} + \frac{1}{Q_{02}}} \quad (6)$$

If in [2, eq. (11)] we introduce the four contributions individually to the lost power previously discussed and evaluated from (9) and (10) of [2], we obtain the expressions for Q_{01} , Q_{02} , Q_{0d} , and Q_{0s} . Correct formulas are obtained only if we introduce in (9) of [2] the complex characteristic impedances Z'_1 and Z'_2 instead of the real characteristic impedances Z_1 and Z_2 valid only for lossless lines

$$Z'_1 = Z_1 \left(1 - j \frac{\alpha_1}{\beta_1} \right), \quad \text{for the evaluation of } Q_{01} \quad (7)$$

$$Z'_2 = Z_2 \left(1 - j \frac{\alpha_2}{\beta_2} \right), \quad \text{for the evaluation of } Q_{02} \quad (8)$$

$$Z'_2 = Z_2 \left(1 + j \frac{\alpha_d}{\beta_2} \right), \quad \text{for the evaluation of } Q_{0d} \quad (9)$$

$$Z'_1 = Z_1, \quad \text{for the evaluation of } Q_{0s}. \quad (10)$$

In (7)–(10), β_1 and β_2 are the phase constants of lines l_1 and l_2 , respectively; α_1 and α_2 the corresponding attenuation constants due to the ohmic losses in the line conductors; $\alpha_d \cong 1/2 \beta_2 \tan \delta$ is the attenuation constant due to the dielectric.

By defining $\theta_1 = \beta_1 \cdot l_1$ and $\theta_2 = \beta_2 \cdot l_2$, the following expressions for Q_{01} , Q_{02} , Q_{0d} , and Q_{0s} are found (a shorter derivation could be used, as shown in [5], which utilizes the

lumped-constants equivalent circuit of the resonator)

$$Q_{01} \cong \frac{\beta_1}{2\alpha_1} \cdot \frac{\frac{2\theta_1}{\sin(2\theta_1)} + \frac{2\theta_2}{\sin(2\theta_2)}}{\frac{2\theta_1}{\sin(2\theta_1)} + 1} \quad (11)$$

$$Q_{02} \cong \frac{\beta_2}{2\alpha_2} \cdot \frac{\frac{2\theta_1}{\sin(2\theta_1)} + \frac{2\theta_2}{\sin(2\theta_2)}}{\frac{2\theta_2}{\sin(2\theta_2)} - 1} \quad (12)$$

$$Q_{0d} \cong \frac{1}{\tan \delta} \cdot \frac{\frac{2\theta_1}{\sin(2\theta_1)} + \frac{2\theta_2}{\sin(2\theta_2)}}{\frac{2\theta_2}{\sin(2\theta_2)} + 1} \quad (13)$$

$$Q_{0s} = Q_{0os} \cdot \frac{2\theta_1 + \frac{2\theta_2}{\sin(2\theta_2)} \cdot \sin(2\theta_1)}{\pi} \quad (14)$$

where Q_{0os} is the value of Q_{0s} for an air-filled $\lambda/4$ resonator

$$Q_{0os} = \frac{\pi}{4} \sqrt{\frac{2 \cdot \sigma_c}{\omega \epsilon_o}} \quad (15)$$

which does not depend on the geometry and dimensions of the resonator.

If in the evaluation of Q_{01} , Q_{02} , and Q_{0d} , only the real components Z_1 and Z_2 of Z'_1 and Z'_2 are introduced in (9) of [2], the following wrong expressions would be found:

$$Q_{01} \cong \frac{\beta_1}{2\alpha_1} \left(1 + \frac{\theta_2}{\theta_1} \frac{\sin(2\theta_1)}{\sin(2\theta_2)} \right) \quad (11')$$

$$Q_{02} \cong \frac{\beta_2}{2\alpha_2} \left(1 + \frac{\theta_1}{\theta_2} \frac{\sin(2\theta_2)}{\sin(2\theta_1)} \right) \quad (12')$$

$$Q_{0d} \cong \frac{1}{\tan \delta} \left(1 + \frac{\theta_1}{\theta_2} \frac{\sin(2\theta_2)}{\sin(2\theta_1)} \right). \quad (13')$$

The values of Q_{01} and Q_{02} are therefore overestimated in (11') and (13'), respectively, while Q_{02} is underestimated in (12'), as discussed in Section IV.

It should be noted that one expects Q_{02} very large when θ_2 is very small. Only the correct (12) confirms this expectation.

The diagrams of Fig. 2 show a normalized Q_0 , i.e., $\eta = Q_0/Q_{0m}$ versus the normalized resonator length x . The reference value Q_{0m} coincides with the value $\beta_1/2\alpha_1$ of Q_{01} for $\theta_1 = \pi/2$ and $\theta_2 = 0$, i.e., for the reference case of a uniform air-filled quarter-wavelength coaxial resonator of characteristic impedance Z_1 and with an ideal lossless short circuit. This reference value has been chosen because it represents generally the highest possible bound to Q_0 for a reference air-filled $\lambda/4$ cavity with an ideal short circuit. Fig. 2 illustrates, for the case $\epsilon_r = 36$, the

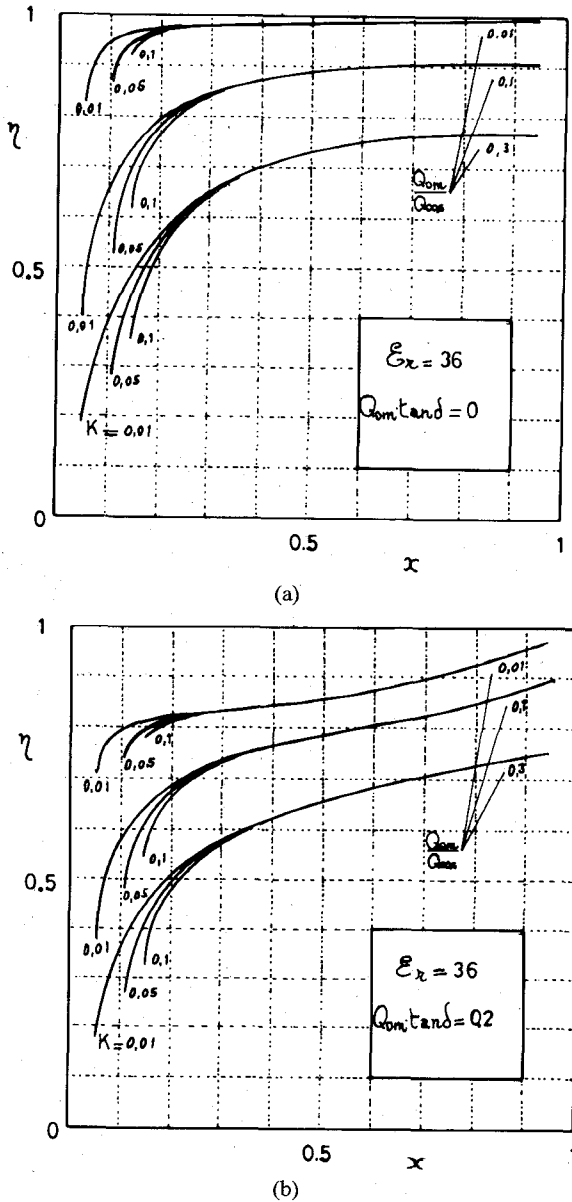


Fig. 2. Behavior of the normalized unloaded Q -factor $\eta = Q_0/Q_{0m}$ versus x , for $\epsilon_r = 36$ at different values of K and Q_{0m}/Q_{0os} (coaxial structure type Fig. 1 with $Z_1 = 76 \Omega$). (a) Case of $\tan \delta = 0$. (b) Case of $Q_{0m} \tan \delta = 0.2$.

influence that $\tan \delta$, ϵ_r , K , Q_{0m}/Q_{0d} , and Q_{0m}/Q_{0os} have on Q_0 for $Z_1 = 76 \Omega$ and the structure of Fig. 1.

While correct equations (11)–(14) are general and do not refer to a particular resonator structure, the diagrams of Fig. 2 show the behavior of Q_0 for the particular case of a coaxial resonator. In fact, in these diagrams, Q_{02} and Q_{01} have been evaluated with reference to the coaxial structure. Examples of diagrams for other resonator structures are presented in [5], which allow for a comparison with the coaxial structure presented here. It should be noted that the influence of $\beta_2/(2 \cdot \alpha_2 \cdot Q_{0m})$ has already been taken into account through the choices of ϵ_r , K , and Z_1 .

The behavior of η is similar for all the values of ϵ_r , i.e., the curves, for any ϵ_r , improve regularly by decreasing K . When K is very small, the curves of η approach the values

obtainable with a uniform impedance air-filled coaxial resonator, tuned with a lumped lossless capacitor C having the same length l_1 and the same characteristic impedance Z_1 . This does not happen with eqs (11'), (12'), and (13').

Regarding the resonator design, the following items should be noted.

For a given value of ϵ_r , K , and θ_1 , only one value of θ_2 exists (and therefore of $l = l_1 + l_2$ and of the length reduction coefficient x), that satisfies the simplified resonance condition [2] which neglects in a first-order approximation the end-effect, and the discontinuity between l_1 and l_2

$$\tan \theta_1 \cdot \tan \theta_2 = K. \quad (16)$$

For a given ϵ_r , a minimum value of x exists [3], [5] if $K < 1/\sqrt{\epsilon_r}$ (curves of Fig. 2 are therefore limited to this minimum value of x).

An optimum value of Z_1 exists, for which Q_0 is maximum; this value depends on K , ϵ_r , θ_1 , and on the geometrical structure of resonator lines l_1 and l_2 ; this value for a low value of K is somewhat higher than the optimum value of Z_1 for the $\lambda/4$ cavity (see Fig. 3).

The loading with high-dielectric constant materials could permit better results than with an air-filled cavity because it is easy to obtain a very low value of K , but only if the $\tan \delta$ of dielectric is sufficiently low compared with $1/Q_{0m}$ (for example with $Q_{0m} \tan \delta = 0.2$ an air-filled cavity could result to be more advantageous).

III. COMPARISON WITH RESULTS THAT NEGLECT THE IMAGINARY COMPONENT OF THE CHARACTERISTIC IMPEDANCE

From (11'), (12'), (13'), and (14), curves of Figs. 3 and 4 (dashed lines) have been evaluated, which show the behavior of η , the normalized Q_0 -factor, i.e., $2\alpha_1 \cdot Q_0/\beta_1$ versus Z_1 (Fig. 3) or versus x (Fig. 4). This normalized choice coincides with that of [2].

These curves have been evaluated for comparison purposes just for the same coaxial resonator, for which the curves of Figs. 2 and 3(a), published in [2], have been calculated, i.e., frequency 900 MHz, $\sigma_c = 5.8 \cdot 10^7 \Omega/\text{m}$, $R_s = 7.827 \cdot 10^{-3}$, $\epsilon_r = 35$, $\tan \delta = 10^{-4}$, outer conductor diameter = 10 mm, $\beta_1/2\alpha_1 = 1240$, $l_2 = 2$ mm (Fig. 3), and $Z_1 = 76 \Omega$ (Fig. 4). The curves published in [2] are very similar to those presented here in Figs. 3 and 4, showing that curves of [2] have been calculated by using only the real parts Z_1 and Z_2 of the characteristic impedances of l_1 and l_2 . For comparative purposes, Figs. 3 and 4 present another set of curves (solid lines) evaluated with correct formulas (11)–(14). The comparison shows the relevant differences between the results obtainable with the correct formulas and those obtainable by neglecting the imaginary parts of Z_1 and Z_2 . It should be noted for example that the wrong curves of the normalized Q -factor versus the normalized resonator length x present an optimum value of K (i.e., $K = 1/\sqrt{\epsilon_r}$), when $\epsilon_r > 1$, which does not exist if the correct calculations are used. This optimum value has not been proved in [2], where the curves of normalized Q were drawn only up to $K = 1/\sqrt{\epsilon_r}$.

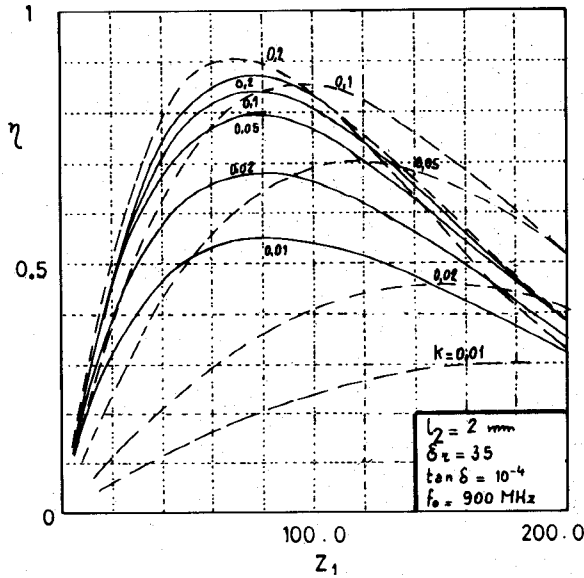


Fig. 3. Behavior of the normalized unloaded Q -factor $\eta = Q_0/Q_{0m}$ versus Z_1 at different values of K (see Fig. 2 of [2]) with $l_2 = 2$ mm, $\epsilon_r = 35$, $\tan \delta = 10^{-4}$, and $f = 900$ MHz. Dashed lines show the curves evaluated with (11'), (12'), (13'), and (14). Solid lines show the curves evaluated with (11)–(14).

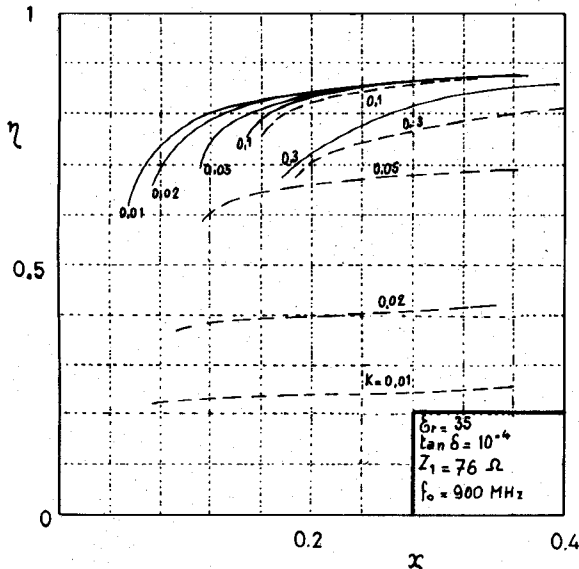


Fig. 4. Behavior of the normalized unloaded Q -factor $\eta = Q_0/Q_{0m}$ versus x at different values of K (see Fig. 3(a) of [2]) with $Z_1 = 76 \Omega$, $\epsilon_r = 35$, $\tan \delta = 10^{-4}$, $f = 900$ MHz. Dashed lines show the curves evaluated with (11'), (12'), (13'), and (14). Solid lines show the curves evaluated with (11)–(14).

As a consequence the inaccurate analysis given by (11'), (12'), and (13') penalizes strongly the choice of a low impedance for l_2 and particularly the case of air-filled l_2 , which requires lower values of K to achieve the same length reduction.

The strong variation exhibited in the dashed curves of Fig. 4 by the optimum value of Z_1 , when K is decreased, is not found in the correct calculations (solid lines), where the corresponding variation is very small.

Finally, it should be observed that the difference between correct and wrong curves is small in the range of values near $K = 1/\sqrt{\epsilon_r}$, which is the range of K values used in the experimental results presented in [2].

IV. REMARKS ON THEORETICAL RESULTS AND COMPARISON WITH MEASURED DATA

The theoretical results presented in this paper are confirmed by the measured data presented in Table I. However, some comments on the reasons why the imaginary component of Z' cannot be neglected in calculations seem useful in order to understand better the comparison with theoretical calculations.

The correct real component R_i of the input impedance of a short-circuited uniform line having propagation constant $\gamma = \alpha + j\beta$ is, as is well known,

$$R_i = \text{Re} \{ Z' \tanh(\gamma l) \} \quad (17)$$

which, in case of ohmic losses, gives results greater than the value R_i obtained by putting Z instead of Z' in (17). For example, in case of $l \ll \lambda/4$, this last value becomes half of the correct first-order value $R_i = 2Z\alpha l$. The contrary behavior is true for an ideal open-circuited uniform line section of length $l < \lambda/4$, having only ohmic losses. In fact, the correct real component G_i of the input admittance

$$G_i = \text{Re} \{ Y' \tanh(\gamma l) \} \quad (18)$$

gives results smaller than the value obtained by neglecting the imaginary component of $Y' = 1/Z'$ in (18). For example, in case of $l \ll \lambda/4$, this last value becomes $G_i = Y\alpha l$, while the correct first-order value is zero.

These considerations clarify why use of Z instead of Z' in (9) of [2] leads one to overestimate ohmic losses in line 2 and to underestimate ohmic losses in line 1, while the stored energies are not modified in first-order approximation. In air-filled resonators, the overall effect is to underestimate Q_0 , when K is small, i.e., when losses in line 2 are greater.

Similar considerations may be made to understand why dielectric losses are underestimated by neglecting the imaginary component of Z' in line 2. This effect may sometimes mask the effect of overestimation of ohmic losses in line 2. The measured data, presented in Table I, confirm these remarks as well the general behavior, observed in the graphs presented in the paper. Table I compares measured data and results of theoretical calculations obtained both with the set of equations (11)–(14) and with the set (11'), (12'), (13'), and (14).

In calculations, a σ value smaller than the value $\sigma = 5.8 \cdot 10^7$ for copper has been used, as a practical value which typically reduces to a few percent the range of discrepancies among measured data and corresponding values calculated with the set of equations (11)–(14). A reduction factor of 0.56 has been assumed for samples 1–5 and 0.62 for the last three samples.

TABLE I
COMPARISON BETWEEN THEORETICAL RESULTS AND
MEASURED DATA

resonator	resonator data							Q_0		
	freq. GHz	Z_1 Ohm	Outer diam. mm	K	x	ϵ_r	$\tan \delta$	measured data	calculated values (11',12',13') (14)	(11',12',13') (14)
1	.9	77	10	.06	.15	35	10^{-4}	751 [2]	754	637
2	.9	77	10	.08	.15	35	10^{-4}	729 [2]	742	692
3	.9	77	10	.105	.15	35	10^{-4}	705 [2]	706	700
4	.9	77	10	.17	.27	35	10^{-4}	809 [2]	812	843
5	.9	77	10	.2	.27	35	10^{-4}	812 [2]	811	834
6	2	76	10	.16	.29	35	$5 \cdot 10^{-4}$	760	800	976
7	2	76	10	.1	.4	1	0	1210	1213	504
8	2	76	10	.05	.4	1	0	1300	1308	313

Measured data on resonators 1–5 of Table I have been taken from [2] in order to show that the behavior of the experimental results of [2] agree with the calculations obtained through the correct set of equations (11) to (14). The remaining measured data have been obtained in the Telettra Laboratories in Milan, Italy.

The first six resonators in Table I use dielectric filling of line 2, while the last two resonators are air filled in order to illustrate the effect of a large range of dielectric permittivities ϵ_r (between 1 and 35).

The first three resonators show that, when K is changed, both correct calculations and measured data present the same behavior, while calculations performed by neglecting the imaginary component of the characteristic impedances present the opposite behavior. In fact, both measured data and correct results of calculations increase by about 7 percent by reducing K from 0.105 to 0.06 with $x = 0.15$; on the contrary, Q_0 values calculated with (11'), (12'), (13'), and (14) decrease of about 9 percent, by giving evidence to the effect of overestimation of ohmic losses in line 2 for low values of K , previously described.

Resonators 4–6, with higher values of K , give evidence to the effect of underestimation of dielectric losses in the calculations performed with (11'), (12'), and (13') with respect both to correct calculations and to measured data. This effect is particularly evident for resonator 6. In fact, a $Q_0 \approx 1290$ would be calculated both with (11)–(14) and (11'), (12'), (13'), and (14) if a lossless dielectric would have been assumed.

The comparison of correct calculations and measured data with the calculations performed with (11'), (12'), (13') and (14) for the air-filled resonators 7 and 8 gives greater evidence that these equations lead to an overestimation of ohmic losses in line 2, when K is very small. In fact, in this case ($\tan \delta = 0$) this effect is not masked by the underestimation of dielectric losses.

V. CONCLUSIONS

The design of resonators for compact microwave bandpass filters using stepped-impedance structures has been reviewed. General formulas have been presented for the unloaded Q -factor of resonators by analyzing separately the various loss contributions to overall Q 's; these formulas allow for a correct design optimization of resonators for any geometrical structure. Curves have also been presented to illustrate the behavior of Q_0 for coaxial resonator parameters and its dependence on various resonator parameters and on the level of miniaturization (i.e., on the length reduction of factor x).

The paper points out the importance, for a correct analysis, of taking into account the imaginary part of the characteristic impedances of lines l_1 and l_2 . A comparison between the results of correct calculations and those obtained by neglecting this imaginary component proves the relevancy of the influence of this component.

The theoretical results presented in this paper have been confirmed by experimental results.

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